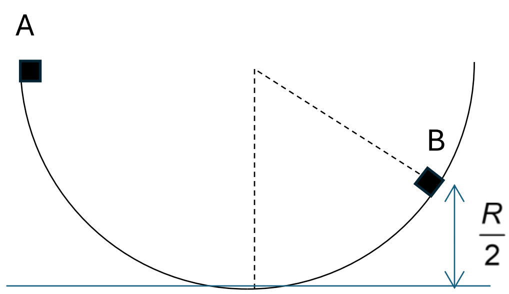
Teacher notes
Topic A

A problem using $W_{ext} = \Delta E_{total}$

A small block of mass *m* is released on the inside of a hemispherical bowl of radius *R*, at A. The block stops instantaneously at B a height $\frac{R}{2}$ from the ground and then slides back down again.



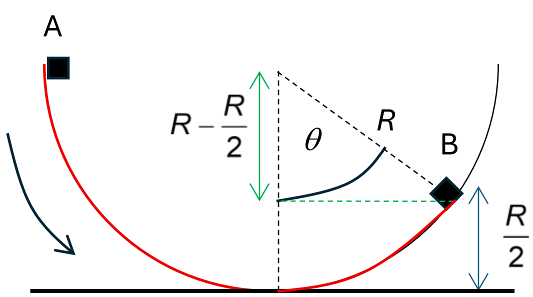
(a) Show that the constant frictional force *f* acting on the block is $f = \frac{3}{5\pi}mg$.

(b) The block slides back down from B. What is the maximum value of the static coefficient of friction between the block and the bowl?

(c) At what height from the ground will the block next come to instantaneous stop?

Answers

(a) The change in the total energy of the ball is equal to the work done by the external forces on the ball, $W_{ext} = \Delta E_{total}$. This change is $mg\frac{R}{2} - mgR = -mg\frac{R}{2}$. The external force doing work is the frictional force f and its work done is $fs\cos 180^\circ = -fs$ where s is the distance travelled.



The distance travelled is shown in red and equals

$$\frac{2\pi R}{4} + R\theta$$

where (see diagram)

$$\cos\theta = \frac{R - \frac{R}{2}}{R}$$

$$= \frac{1}{2}$$

so $\theta = \frac{\pi}{3}$. Hence $s = \frac{\pi R}{2} + \frac{\pi R}{3}$

$$-mg\frac{R}{2} = -f\left(\frac{\pi R}{2} + \frac{\pi R}{3}\right)$$

And so

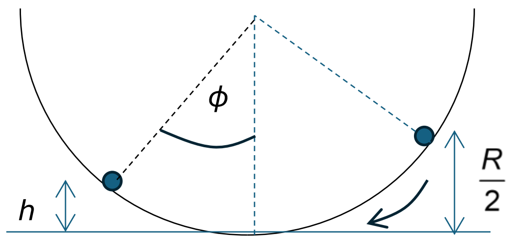
$$f = \frac{3}{5\pi}mg$$

(b) The maximum static frictional force that can develop at B is $\mu_s N = \mu_s mg\cos 60^\circ = \frac{\mu_s mg}{2}$. This has to be smaller than $mg\sin 60^\circ = \frac{mg\sqrt{3}}{2}$. Hence

$$\frac{\mu_s mg}{2} < \frac{mg\sqrt{3}}{2}$$

implying that $\mu_s < \sqrt{3}$.

(c) Applying $W_{ext} = \Delta E_{total}$ again we find



$mg\frac{R}{2} - mgh = -f(R\frac{\pi}{3} + R\phi)$ where $\cos\phi = \frac{R-h}{R}$.

So, we have

$$mgh - mg\frac{R}{2} = -\frac{3}{5\pi}mgR(\frac{\pi}{3} + \cos^{-1}\frac{R-h}{R})$$

$$\frac{h}{R} = \frac{1}{2} - \frac{1}{5} - \frac{3}{5\pi}\cos^{-1}(1 - \frac{h}{R})$$

$$\frac{h}{R} = \frac{3}{10} - \frac{3}{5\pi}\cos^{-1}(1 - \frac{h}{R})$$

Using the GDC, the solution is $\frac{h}{R} \approx 0.183$, i.e. $h \approx 0.183R$.

Note: It is easy to check that the block does not stop **before** getting to the bottom of the bowl.